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# DYNAMICAL GENERATION OF THE CKM MATRIX

by

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## Abstract

We argue that in dynamical models of fermion masses, which explain the three mass scales of the generations of fermions with three separate heavy scales above the electroweak symmetry breaking scale, the off diagonal CKM matrix elements must be generated at a fourth scale. The simplest form for the additional mass generation is a mechanism that has an  $SU(3)_L \otimes SU(3)_R$  family symmetry. We show that such an ansatz can produce the up down mass inversion and the observed Cabibbo angle. If the mass terms for the third family quarks are enhanced by additional interactions the full CKM matrix may be realized. We present a toy model which realizes vacua with these masses.

Recent work has shown that one family extended technicolour (ETC) models [1] may be built that are both plausibly consistent with the precision data from LEP [2] and compatible with the third family masses [3]. These models have a techni-family mass spectrum of the form

$$M_U = M_D \sim 400 GeV, \quad M_E \sim 150 GeV, \quad M_N \sim 50 GeV \quad (1)$$

In Ref[3] the author has argued that if there is a single feed down ETC interaction for each of the first and second families then those families' masses (neglecting neutrinos) follow naturally from this spectrum and the large top mass (generated by a direct, but sub-critical, top condensating ETC interaction). The analysis of Ref[3] does not however generate the up down mass inversion or the off diagonal CKM matrix elements. With the notable exception of Refs[4, 5, 6] there has been little discussion of the CKM matrix generation in dynamical models in the literature. In this letter we propose a simple form for the off diagonal mass terms and show that it is compatible with the observed CKM matrix elements. In addition the ansatz is simultaneously capable of generating the up down mass inversion. Finally we present a toy model with vacua that generate these mass matrices. A naive analysis of the effective potential in the model does not however favour these precise vacua.

ETC models generate the light fermion masses by the exchange of heavy gauge bosons which couple the light fermions to techni-fermion condensates with coupling  $g^2/M_{ETC}^2$ . The hierarchy of three families may either be generated by three separate magnitude couplings at one ETC mass scale or by three ETC mass scales. Though the former possibility may be realized in models with complicated mixings and/or many ETC gauge groups (eg Ref[4]) we consider the latter with a single ETC group for the entire techni-family more natural. However, if the ETC breaking is of the form

$$SU(N+3)_{ETC} \xrightarrow{\Lambda_1} SU(N+2) \xrightarrow{\Lambda_2} SU(N+1) \xrightarrow{\Lambda_3} SU(N)_{TC} \quad (2)$$

then the gauge eigenstates corresponding to the light three families are strictly defined by the breaking pattern. The mass eigenstates are identical to the gauge eigenstates and the model can not give rise to relative rotations between the up and down type quark mass matrices. Additional dynamics at a fourth (or more) scale is required to generate the CKM matrix elements.

In order to show how off diagonal fermion mass terms may be generated let us first review the vacuum structure of QCD. We shall just consider the QCD interactions of the up and down quarks. The theory has an  $SU(2)_L \otimes SU(2)_R$  flavour symmetry that is broken by the current quark masses

$$M_c = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \quad (3)$$

This breaking term allows us to distinguish a number of distinct vacua described by the condensates

$$(\bar{u}_L, \bar{d}_L) U \begin{pmatrix} u_R \\ d_R \end{pmatrix} \quad (4)$$

where  $U$  is an element of  $SU(2)$ . Clearly except when  $U$  is the identity these vacua in addition to the current masses give off diagonal dynamical masses. In QCD the current masses perturb the energy of each of these vacua through the contributions to the effective potential of the form

$$- Tr(M_c^\dagger U) + h.c. \quad (5)$$

generated by the diagram in Fig1. The lowest energy vacuum is that with  $U$  equal to the identity (which maximises the trace) and no off diagonal terms are observed. This vacuum is also preferred since it leaves the  $U(1)$  of QED unbroken.

We propose that a similar vacuum structure could give rise to the CKM matrix mixings in technicolour models though the potential would have to take a different form.

### An Ansatz With $SU(3)$ Family Symmetry

The simplest assumption about the dynamics responsible for the CKM elements is that it is  $SU(3)_L \otimes SU(3)_R$  family symmetric and flavour blind. Our ansatz for the additional contribution to the up and down type quark mass matrices are thus proportional to the identity in some basis rotated by an element of  $SU(3)$  relative to the usual ETC interactions in Eqn(2). We have

$$\begin{aligned} M_U &= \begin{pmatrix} m_t^{ETC} & 0 & 0 \\ 0 & m_c^{ETC} & 0 \\ 0 & 0 & m_u^{ETC} \end{pmatrix} + \tilde{m}U \\ M_D &= \begin{pmatrix} m_b^{ETC} & 0 & 0 \\ 0 & m_s^{ETC} & 0 \\ 0 & 0 & m_d^{ETC} \end{pmatrix} + \tilde{m}U \end{aligned} \tag{6}$$

where  $\tilde{m}$  is the mass scale generated by the new dynamics,  $m_i^{ETC}$  is the diagonal mass generated by the usual ETC interactions and  $U$  is an element of  $SU(3)$ . Following Ref[5] we parameterize  $U$  in terms of an  $SU(2)$  matrix,  $\Sigma$ , and phases

$$U = \begin{pmatrix} e^{-i\phi}(1 - (1 - \cos\theta)uu^\dagger)\Sigma & -\sin\theta u \\ \sin\theta u^\dagger\Sigma & \cos\theta e^{i\phi} \end{pmatrix} \tag{7}$$

with  $u$  a unimodular 2-vector

$$u = \begin{pmatrix} ae^{i\alpha} \\ (1-a^2)^{\frac{1}{2}}e^{i\beta} \end{pmatrix} \quad (8)$$

The diagonal mass matrices are given by

$$\begin{aligned} M_U^{diag} &= L_U^\dagger M_U R_U \\ M_D^{diag} &= L_D^\dagger M_D R_D \end{aligned} \quad (9)$$

where  $L_i$  and  $R_i$  are left and right handed SU(3) family transformations that diagonalize the hermitian matrices  $M_i^\dagger M_i$  and  $M_i M_i^\dagger$  respectively. The CKM matrix is then given by

$$K = L_D^\dagger L_U \quad (10)$$

## The Cabibbo Sector

For simplicity we first consider numerical results for only the lightest two families of quarks. The CKM matrix phases may then be rotated away by global family transformations. Standard ETC interactions do not naturally generate the up down mass inversion (preferring in fact the inverse) or CKM mixings. We shall therefore take as our ansatz

$$M_U = \begin{pmatrix} 1.5 & 0 \\ 0 & 0.005 \end{pmatrix} + \tilde{m}_U \text{ GeV} \quad (11)$$

$$M_D = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.002 \end{pmatrix} + \tilde{m}_D \text{ GeV} \quad (12)$$

Mixing angles will only result from off diagonal mass terms (on diagonal masses simply correspond to rescaling the ETC feed down masses) and therefore we take as our SU(2) matrix

$$U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (13)$$

and search for values of  $\tilde{m}$  compatible with the observed masses and mixings. We find for  $\tilde{m} = 0.057 GeV$

$$\begin{matrix} m_c = 1.5 GeV & m_s = 0.2 GeV \\ m_u = 0.003 GeV & m_d = 0.013 GeV \end{matrix} |K| = \begin{pmatrix} 0.975 & 0.22 \\ 0.22 & 0.975 \end{pmatrix} \quad (14)$$

The ansatz correctly describes both the up down mass inversion and the Cabibbo sector of the CKM matrix.

### Three Families

In extending the ansatz to the third family we shall again concentrate on the CKM matrix magnitudes since the CKM matrix phase is ill determined. U now has three parameters ( $a, \theta$  and its equivalent angle in  $\Sigma$ ) that determine the magnitudes of the CKM elements and six phases. Solutions again exist for this ansatz that are compatible with the up and down quark masses and the Cabibbo angle. For example

$$\begin{aligned} M_U &= \begin{pmatrix} 170 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 0.005 \end{pmatrix} + 0.052U \text{ GeV} \\ M_D &= \begin{pmatrix} 5 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.002 \end{pmatrix} + 0.052U \text{ GeV} \end{aligned} \quad (15)$$

where

$$|U| = \begin{pmatrix} 0.76 & 0.42 & 0.63 \\ 0.41 & 0.28 & 0.87 \\ 0.48 & 0.86 & 0.04 \end{pmatrix} \quad (16)$$

diagonalizes to give

$$\begin{array}{ll}
m_t = 170\text{GeV} & m_b = 5\text{GeV} \\
m_c = 1.5\text{GeV} & m_s = 0.2\text{GeV} \\
m_u = 0.007\text{GeV} & m_d = 0.013\text{GeV}
\end{array}
|K| = \begin{pmatrix} 0.9999 & 0.006 & 0.003 \\ 0.005 & 0.975 & 0.22 \\ 0.004 & 0.22 & 0.975 \end{pmatrix} \quad (17)$$

We note that these numerical results are not the result of any fine tuning in the ansatz's parameter space but are stable over a range of parameters other than the requirement that  $|U_{33}|$  is at least an order of magnitude suppressed relative to the other elements of U. The third family mixing angles can be raised to their observed values by increasing  $\tilde{m}$  but at the expense of increasing the up and down quark masses above their experimental limits.

The third family mass elements may be enhanced relative to the other families' mass terms if the ETC model has strong extended technicolour interactions. Ref[3] makes use of strong horizontal self interactions in the techni and third families to break the SU(8) flavour symmetry in the pattern observed in the light fermion masses. The simplest realization of this global symmetry breaking would be through the addition of an extra strong (but not super-critical) broken U(1) gauge interaction with appropriate couplings. If such a gauge boson mixed with the standard ETC gauge boson associated with the diagonal generator in the breaking  $SU(N+1)_{ETC} \rightarrow SU(N)_{TC}$  then the self coupling of the third generation would naturally be greater than that of the first and second generations. It is then plausible that the third family quark mass elements may be enhanced relative to those of the lighter two families. Parameter ranges then exist that reproduce the full CKM matrix, for example with U in Eqn(15)

$$|U| = \begin{pmatrix} 0.92 & \zeta 0.38 & \zeta 0.09 \\ \zeta 0.07 & 0.07 & 0.99 \\ \zeta 0.34 & 0.92 & 0.04 \end{pmatrix} \quad (18)$$

and with  $\zeta = 10$  (note that when  $\zeta = 1$  U is unitary and that a parameter multiplying the on diagonal third family mass terms simply corresponds to a rescaling of the ETC masses in Eqn(15)) we obtain

$$\begin{array}{ll}
m_t = 170 GeV & m_b = 5 GeV \\
m_c = 1.5 GeV & m_s = 0.2 GeV \\
m_u = 0.007 GeV & m_d = 0.014 GeV
\end{array}
|K| = \begin{pmatrix} 0.9993 & 0.04 & 0.003 \\ 0.04 & 0.975 & 0.22 \\ 0.003 & 0.22 & 0.975 \end{pmatrix} \quad (19)$$

In a realistic model we might expect different  $\zeta$ s in the up and down type quark mass sectors (corresponding to the difference in top bottom mass splitting) however this would simply add new parameters to the ansatz. We therefore expect that solutions fitting the observed CKM matrix element values would still exist. We note that this ansatz is not postdictive having the same number of input and output parameters. Nevertheless it is interesting that this simple ansatz is viable.

## A Toy Model

In this section we introduce a simple model that realizes vacua with the mass matrices discussed above. The model has a full family of fermions that transform under the usual ETC group  $SU(N+3)_{ETC}$  plus some assumed additional dynamics that break the  $SU(8)$  flavour symmetry of the fermion family. The simplest additional dynamics would be, as discussed above, simply to add in a new  $U(1)$  gauge interaction with different charges for each of the fermion flavours. We shall assume that the standard ETC dynamics and this additional sector give rise to the diagonal contributions to the fermion masses in Eqn(6) and concentrate our discussion on the generation of the extra mass contributions which are family symmetric.

In addition to these usual dynamics the flavours of fermions that mix (including at least all the quarks) transform under some  $SU(M+1)$  gauge group which we shall call ultracolour. This additional gauge group breaks at some high scale,  $\Lambda_U$ , according to

$$SU(M+1)_{UC} \rightarrow SU(M) \quad (20)$$

The broken singlet forms the usual quark and techni-quark sectors. Whether an additional



unification with  $SU(3)$  colour occurs we leave for future model builders. The ultracolour group then becomes confining, generating dynamical masses for the ultracoloured fermions which feed down to the singlets through the gauge bosons that acquired masses at the scale  $\Lambda_U$ . The ultracoloured partners of the usual quarks have an  $SU(3)_L \otimes SU(3)_R$  family symmetry under the ultracolour interactions and will thus give rise to vacua with mixings between the families. These mixings will feed down to the standard quark sector producing masses that mix the families as described in Eqn(6). This dynamics is entirely analogous to that in QCD with the ETC fed down masses playing the part of the current quark masses in Eqn(3). Presumably the effective potential will therefore take the same form as that in Eqn(5) (again generated by the diagram in Fig 1 but with ultra-quarks in the loop) and the vacua corresponding to the ansatz in Eqn(18) would not be the preferred one. However, it is possible that dynamics associated with the high breaking scales may introduce additional parameters that perturb the effective potential resulting in the realistic vacua. It is not our intention in this letter to propose a phenomenologically consistent model but merely provide an existence proof of models which potentially give rise to CKM mixing. In this spirit also we shall not be troubled by our need to reconcile the extra  $M \times (N + 3)$  ultracoloured quark doublets with measurements of the precision parameter  $S$  [2] that prefer to minimize the number of strongly interacting doublets.

In this letter we have proposed that a family symmetric contribution to the quark mass matrices may be generated in ETC models at an additional scale to the usual ETC breaking scales. Such an ansatz is capable of explaining the up down mass inversion simultaneously with the Cabibbo angle. If the third family quark mass terms are enhanced relative to those of the lighter two families the ansatz can describe the full CKM matrix structure. We have discussed a toy technicolour model that possesses vacua consistent with our ansatz for the quark masses. The model serves to highlight the problem that the naive effective potential of such a model does not favour the precise physical vacuum. Nevertheless we hope that this

discussion sheds light on the form of the CKM matrix and will be helpful in future model building.

### **Acknowledgements**

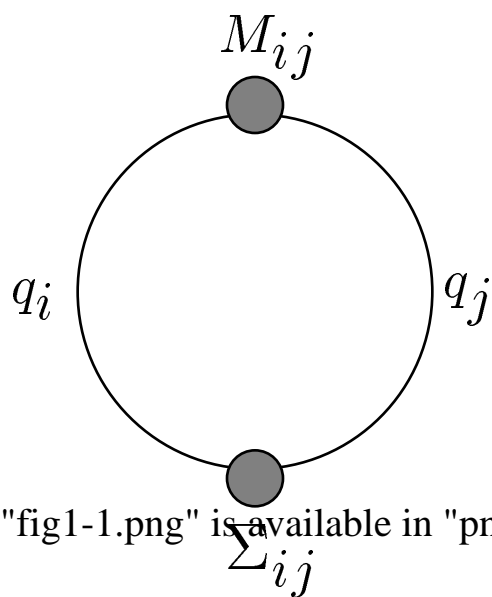
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### **Figure Caption**

Fig1: Contribution to the effective potential from (ultra) quark loops.

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Fig 1